

Determinantlar nazariyasining elementlari

Mazkur bobda oily algebraning determinant tushunchasini va uning chiziqli tenglamalar sistemasini yechishga tatbiqini keltiramiz.

2.1. Determinant

Faraz qilaylik, $\{a_{ij} : i = \overline{1;n}, j = \overline{1;n}, n \in N\}$ to'plam berilgan bo'lib, uning elementlari bo'lgan a_{ij} ifodalar ustida arifmetik amallar aniqlangan bo'lsin.

2.1.1-ta'rif. Yuqorida berilgan to'plamning har bir elementini *birinchi tartibli determinant* deb ataymiz va uning qiymati sifatida shu elementning o'zini qabul qilamiz ([4] §16).

Endi, $(n-1)$ – tartibli determinant tushunchasi aniqlangan degan faraz asosida n -tartibli determinantni ($2 \leq n \in N$)

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} \quad (2.1.1)$$

kabi belgilab, uning qiymatini aniqlash maqsadida quyidagi yordamchi tushunchalarni kiritamiz.

1) Yotiq (gorizontal) ko'rinishda yozilgan $(a_{i1}, a_{i2}, \dots, a_{in})$ ni (2.1.1) *determinantning i-satri* (*i-satr – vektori*);

2) tik (vertikal) ko'rinishda yozilgan

$$\begin{pmatrix} a_{1j} \\ a_{2j} \\ \dots \\ a_{nj} \end{pmatrix}$$

ni *j-ustuni* (*j-ustun – vektori*);

3) $(a_{11}, a_{22}, \dots, a_{nn})$ ni esa *diagonali* (*bosh diagonali*) deb ataymiz.

2.1.2-ta'rif. $2 \leq n \in N$ bo'lganda (2.1.1) n -tartibli determinant a_{ij} ($i = \overline{1;n}, j = \overline{1;n}$) elementining $(n-1)$ - *tartibli minori* (*minori*) deb, mazkur element joylashgan determinantning *i-satri* va *j-ustunini* o'chirish natijasida hosil qilingan $(n-1)$ - tartibli determinantni ataymiz hamda M_{ij} kabi belgilaymiz.

Bu ta'rifdan ko'rinadikki, n - tartibli determinantda n^2 ta $(n-1)$ - tartibli minorlar mavjud bo'lib, yuqoridagi farazga ko'ra, ularning barchasi aniqlangandir.

2.1.3- ta'rif. (2.1.1) n - tartibli determinant a_{ij} elementining *algebraik to'ldiruvchisi* deb, $(-1)^{i+j} \cdot M_{ij}$ ni ataymiz va A_{ij} bilan belgilaymiz.

Demak, bu ta'rifga asosan,

$$A_{ij} = (-1)^{i+j} \cdot M_{ij} \quad (i = \overline{1;n}, j = \overline{1;n}). \quad (2.1.2)$$

2.1.4-ta'rif. n -tartibli determinantning qiymati deb

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} = \sum_{j=1}^n a_{1j} A_{1j} \quad (2.1.3)$$

ni qabul qilamiz ([4] §16).

(2.1.3) ning o'ng tomonini *determinantning birinchi satri bo'yicha yoyilmasi* deb yuritiladi.

2.1.1. Ikkinchi tartibli determinant

Agar (2.1.3) da $n=2$ deb olsak, 2.1.1-ta'rif bilan birinchi tartibli determinant tushunchasi aniqlanganligi sababli, (2.1.2) yordamida

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \sum_{j=1}^2 a_{1j} A_{1j} = a_{11} M_{11} - a_{12} M_{12} = a_{11} a_{22} - a_{12} a_{21},$$

ya'ni

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} a_{22} - a_{12} a_{21} \quad (2.1.4)$$

ni olamiz.

Ikkinchi tartibli determinantda (a_{11}, a_{22}) *diagonal* bilan bir qatorda (a_{12}, a_{21}) ni *yordamchi diagonal* deb yuritiladi.

(2.1.4) dan *ikkinchi tartibli determinant qiymati diagonal elementlari ko'paytmasidan yordamchi diagonal elementlari ko'paytmasini ayirish natijasiga tengligini ko'ramiz.*

1-misol. $\begin{vmatrix} -1 & 5 \\ -2 & 6 \end{vmatrix} = (-1) \cdot 6 - (-2) \cdot 5 = -6 + 10 = 4.$

2-misol.

$$\begin{vmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{vmatrix} = \sin \alpha \cdot \sin \alpha - \cos \alpha \cdot (-\cos \alpha) = \sin^2 \alpha + \cos^2 \alpha = 1.$$

3-misol. $\begin{vmatrix} \operatorname{tg} \alpha & -1 \\ 1 & \operatorname{tg} \alpha \end{vmatrix} = \operatorname{tg} \alpha \cdot \operatorname{tg} \alpha - 1 \cdot (-1) = \operatorname{tg}^2 \alpha + 1 = \frac{1}{\cos^2 \alpha}.$

4-misol.

$$\begin{vmatrix} \cos \alpha & \sin \alpha \\ -\rho \sin \alpha & \rho \cos \alpha \end{vmatrix} = \rho \cos^2 \alpha + \rho \sin^2 \alpha = \rho (\cos^2 \alpha + \sin^2 \alpha) = \rho.$$

2.1.2. Uchinchi tartibli determinant

$n=3$ bo'lganda, ikkinchi tartibli determinant (2.1.4) bilan aniqlanganligini hisobga olib, (2.1.3) dan

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} -$$

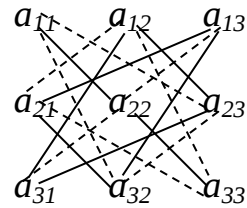
$$- a_{13} a_{22} a_{31} - a_{12} a_{21} a_{33} - a_{11} a_{23} a_{32} \quad (2.1.5)$$

ni olish qiyin emas. Bu uchinchi tartibli determinantni hisoblashning asosiy formulasi bo'lib, uning o'ng tomonidagi ifodani hosil qilishda (2.1.3) bilan bir qatorda *uchburchaklar* hamda *Sarius qoidasi* deb ataluvchi usullar mavjuddir.

1. *Uchburchaklar qoidasi*. Asosi uchinchi tartibli determinant diagonaliga parallel uchlari esa determinant elementlari joylashgan nuqtalarda bo'lgan teng yonli uchburchaklar chizamiz (tasavvurimizda). Diagonalda joylashgan va aytilgan uchburchaklar uchlarida joylashgan 3 tadan elementlarni ko'paytirib, (2.1.5) ning o'ng tomonidagi ifodaning dastlabki 3 ta hadini, xuddi shunday ishni (a_{13}, a_{22}, a_{31}) yordamchi diagonal uchun ham takrorlab, olingan ko'paytmalarni qarama-qarshi ishoralar bilan olib, so'nggi 3 ta hadini hosil qilamiz (2.1.1-rasm).

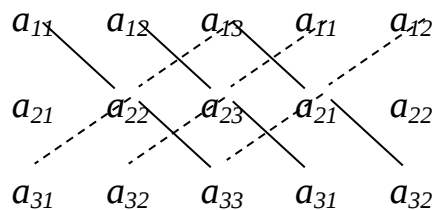
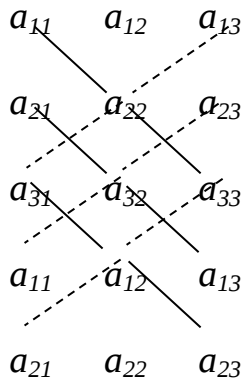
5-misol.

$$\begin{vmatrix} 2 & -1 & 0 \\ 3 & -4 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 2 \cdot (-4) \cdot (-1) + (-1) \cdot 1 \cdot 2 + 3 \cdot 1 \cdot 0 - \\ - 2 \cdot (-4) \cdot 0 - (-1) \cdot 3 \cdot (-1) - 1 \cdot 1 \cdot 2 = \\ = 8 - 2 + 0 - 0 - 3 - 2 = 1.$$



2.1.1-rasm.

2. *Sarius qoidasi*. Bu usulda uchinchi tartibli determinantning birinchi va ikkinchi satrlarini uning davomiga mos ravishda to'rtinchi va beshinchi satrlar qilib yozib, determinant diagonaliga parallel uchtdan elementlar orqali o'tuvchi yana ikkita diagonal o'tkazamiz. Ularga joylashgan uchtdan elementlarni ko'paytirib (2.1.5) ning birinchi uchta hadini va bu ishni yordamchi diagonal uchun ham bajarib olingan ko'paytmalar ishorasini qarama-qarshisiga o'zgartirib, so'ngi uchta hadini olamiz. Huddi shunga o'xshash ishni determinantning birinchi va ikkinchi ustunlarini mos ravishda to'rtinchi va beshinchi ustunlar qilib yozish bilan ham bajarish mumkin (2.1.2-rasm).



2.1.2-rasm.